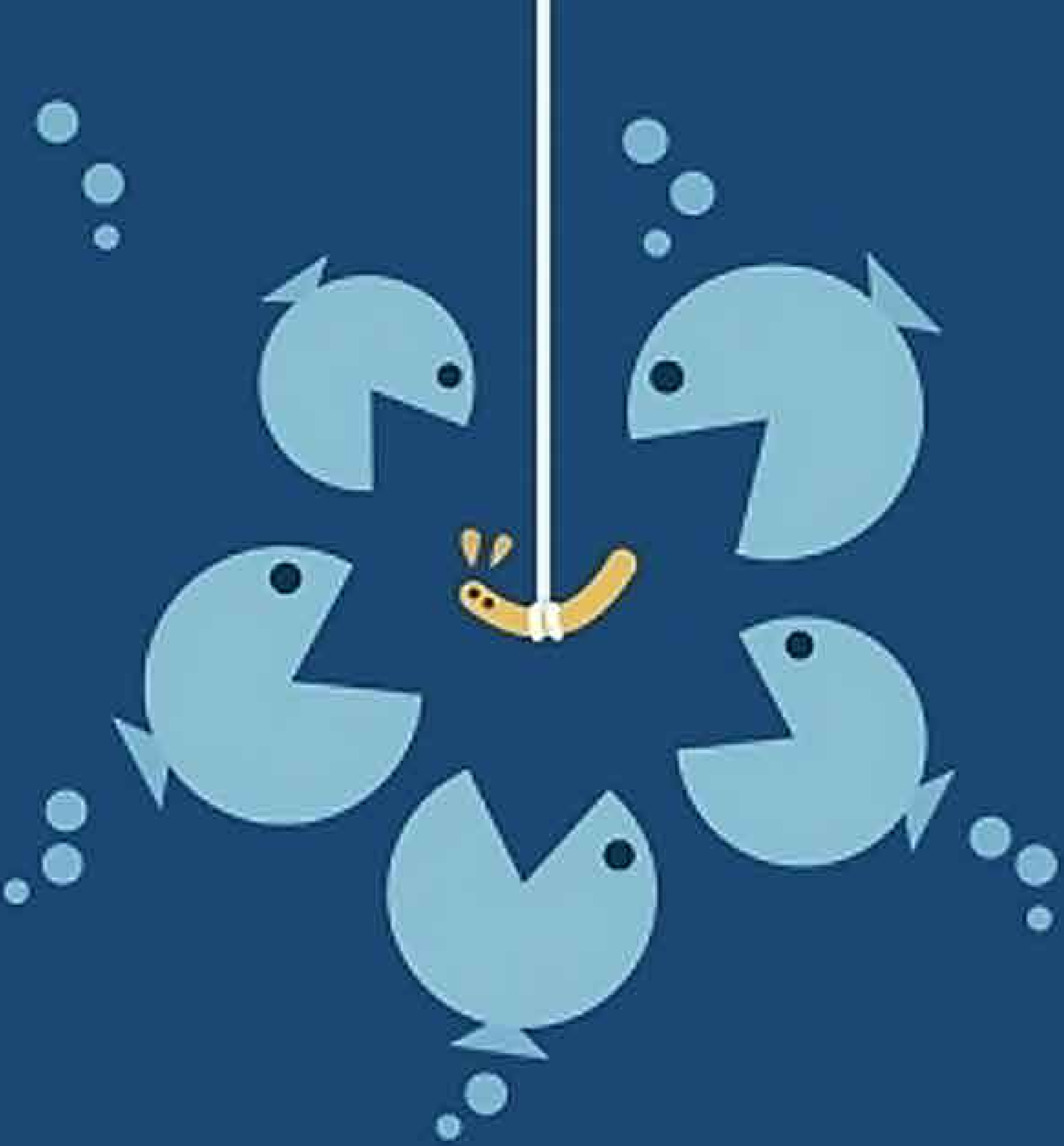
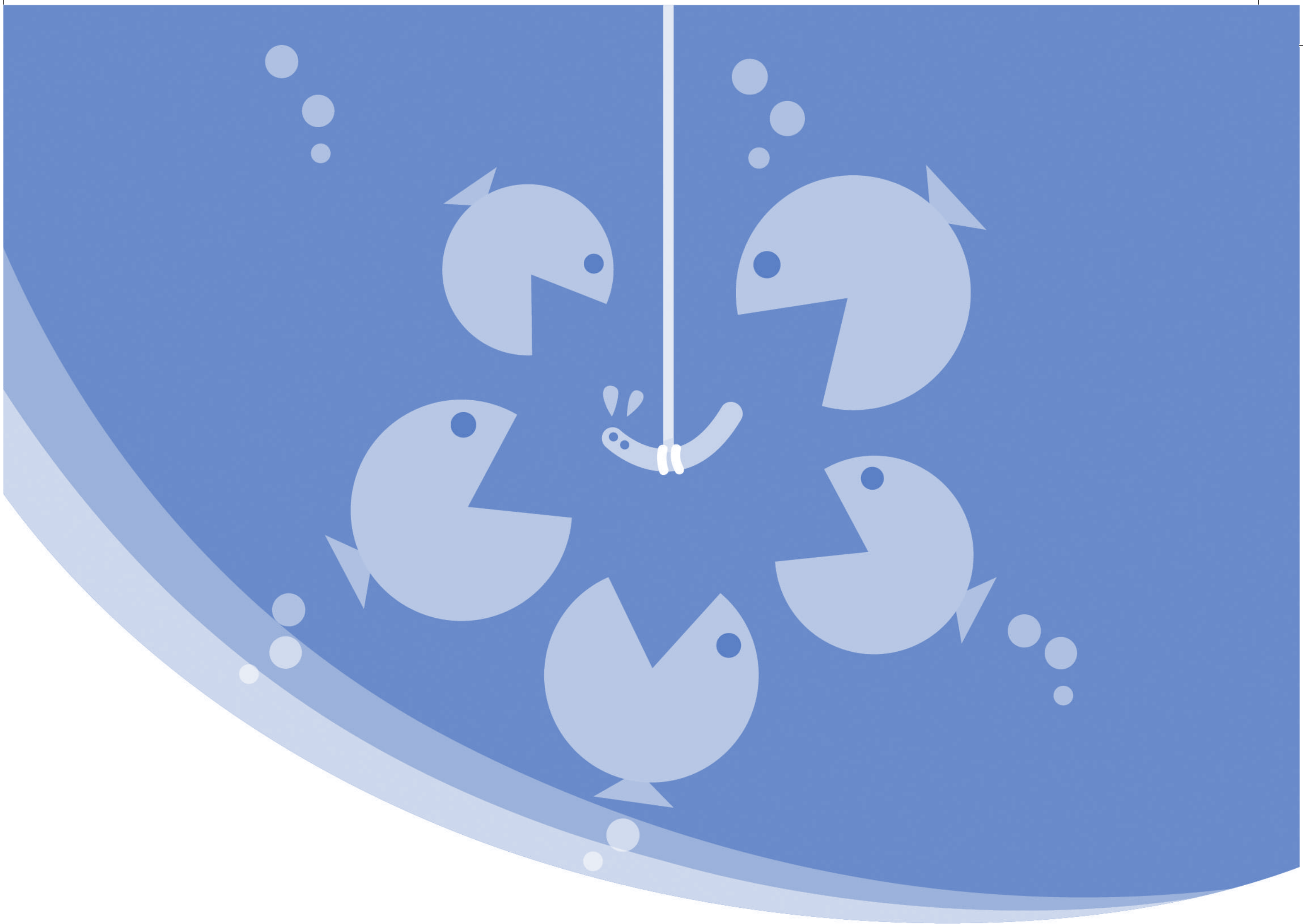


CHRISTOPHER SNYDER, WALTER NICHOLSON, ROBERT STEWART

MICROECONOMIC THEORY

BASIC PRINCIPLES AND EXTENSIONS

EUROPE, MIDDLE EAST & AFRICA EDITION



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PREFACE

This first Europe, Middle East and Africa adaptation of the 11th edition of *Microeconomic Theory: Basic Principles and Extensions* represents a significant effort to continue refining and modernising our treatment of microeconomics. The text retains all of the elements that have made it successful for so many editions, including the basic approach being a focus on building intuition about economic models while providing students with the mathematical tools needed to go further in their studies. The text also seeks to facilitate that linkage by providing many numerical examples, advanced problems and extended discussions of empirical implementation – all of which are intended to show students how microeconomic theory is used today. New developments continue to keep the field exciting, and we hope this edition manages to capture that excitement.

To tailor the text better to the course structures and teaching needs of the Europe, Middle East and Africa market, this new edition adapted by Robert Stewart, an Economics lecturer based in Johannesburg, South Africa, includes the following changes:

- Some content reordering, restructuring and rewriting, to more closely fit typical EMEA course outlines, including:
  - Merging of former Chapter 9 (‘Production Functions’) and Chapter 10 (‘Cost Functions’) into single Chapter 9, ‘Production and Cost Functions’.
  - Content changes within Part Six to be newly titled ‘Factor Market Pricing’ (instead of ‘Pricing in Input Markets’).
- Revisions to end-of-chapter Problems and Analytical Problems.
- A streamlined book encouraging blended print and digital learning, with Chapter Extensions, Brief Answers to Queries and Solutions to Odd-Numbered Problems now on the companion website.
- Complete conversion from Imperial to metric, for all units of measure.
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- New text design and appearance.

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PART 1

INTRODUCTION

This part contains two chapters. Chapter 1 examines the general philosophy of how economists build models of economic behaviour. Chapter 2 then reviews some of the mathematical tools used in the construction of these models. The mathematical tools from Chapter 2 will be used throughout the remainder of this book.

1 ECONOMIC MODELS

The main goal of this book is to introduce you to the most important models that economists use to explain the behaviour of consumers, firms and markets. These models are central to the study of all areas of economics. Therefore, it is essential to understand both the need for such models and the basic framework used to develop them. The goal of this chapter is to begin this process by outlining some of the conceptual issues that determine the ways in which economists study practically every question that interests them.

THEORETICAL MODELS

A modern economy is a complicated entity. Thousands of firms engage in producing millions of different goods. Many millions of people work in all sorts of occupations and make decisions about which of these goods to buy. Let's use peanuts as an example. Peanuts must be harvested at the right time and shipped to processors who turn them into peanut butter, peanut oil, peanut brittle and numerous other peanut delicacies. These processors, in turn, must make certain that their products arrive at thousands of retail outlets in the proper quantities to meet demand.

Because it would be impossible to describe the features of even these peanut markets in complete detail, economists have chosen to abstract from the complexities of the real world and develop rather simple models that capture the 'essentials'. Just as a road map is helpful even though it does not record every house or every store, economic models of, say, the market for peanuts are also useful even though they do not record every minute feature of the peanut economy. In this book we will study the most widely used economic models. We will see that, even though these models often make heroic abstractions from the complexities of the real world, they nonetheless capture essential features that are common to all economic activities.

The use of models is widespread in the physical and social sciences. In physics, the notion of a 'perfect' vacuum or an 'ideal' gas is an abstraction that permits scientists to study real-world phenomena in simplified settings. In chemistry, the idea of an atom or a molecule is actually a simplified model of the structure of matter. Architects use mock-up models to plan buildings. Television repairers refer to wiring diagrams to locate problems. Economists' models perform similar functions. They provide simplified portraits of the way individuals make decisions, the way firms behave, and the way in which these two groups interact to establish markets.

VERIFICATION OF ECONOMIC MODELS

Of course, not all models prove to be 'good'. For example, the earth-centred model of planetary motion devised by Ptolemy was eventually discarded because it proved incapable of accurately explaining how the planets move around the sun. An important purpose of scientific investigation is to sort out the 'bad' models from the 'good'. Two general methods have been used for verifying economic models: (1) a direct approach, which seeks to establish the validity of the basic assumptions on which a model is based; and (2) an indirect approach, which attempts to confirm validity by showing that a simplified model correctly predicts real-world events. To illustrate the basic differences between the two approaches, let's briefly examine a model that we will use extensively in later chapters of this book – the model of a firm that seeks to maximise profits.

The Profit-Maximisation Model

The model of a firm seeking to maximise profits is obviously a simplification of reality. It ignores the personal motivations of the firm's managers and does not consider conflicts among them. It assumes that profits are the only relevant goal of the firm; other possible goals, such as obtaining power or prestige, are treated as unimportant. The model also assumes that the firm has sufficient information about its costs and the nature of the market to which it sells to discover its profit-maximising options. Most real-world firms, of course, do not have this information readily available. Yet such shortcomings in the model are not necessarily serious. No model can exactly describe reality. The real question is whether this simple model has any claim to being a good one.

Testing Assumptions

One test of the model of a profit-maximising firm investigates its basic assumption: do firms really seek maximum profits? Some economists have examined this question by sending questionnaires to executives, asking them to specify the goals they pursue. The results of such studies have been varied. Businesspeople often mention goals other than profits or claim they only do 'the best they can' to increase profits given their limited information. On the other hand, most respondents also mention a strong 'interest' in profits and express the view that profit maximisation is an appropriate goal. Therefore, testing the profit-maximising model by testing its assumptions has provided inconclusive results.

Testing Predictions

Some economists, most notably Milton Friedman, deny that a model can be tested by inquiring into the 'reality' of its assumptions.[1] They argue that all theoretical models are based on 'unrealistic' assumptions; the very nature of theorising demands that we make certain abstractions. These economists conclude that the only way to determine the validity of a model is to see whether it is capable of predicting and explaining real-world events. The ultimate test of an economic model comes when it is confronted with data from the economy itself.

Friedman provides an important illustration of that principle. He asks what kind of theory one should use to explain the shots expert pool players will make. He argues that the laws of velocity, momentum and angles from theoretical physics would be a suitable model. Pool players shoot shots as *if* they follow these laws. But most players asked whether they precisely understand the physical principles behind the game of pool will undoubtedly answer that they do not. Nonetheless, Friedman argues, the physical laws provide accurate predictions and therefore should be accepted as appropriate theoretical models of how experts play pool.

Thus, a test of the profit-maximisation model would be provided by predicting the behaviour of real-world firms by assuming that these firms behave *as if* they were maximising profits. (See Example 1.1 later in this chapter.) If these predictions are reasonably in accord with reality, we may accept the profit-maximisation hypothesis. However, we would reject the model if real-world data seem inconsistent with it. Hence the ultimate test of any theory is its ability to predict *real-world events*.

Importance of Empirical Analysis

The primary concern of this book is the construction of theoretical models. But the goal of such models is always to learn something about the real world.[2]

---

[1] See M. Friedman, *Essays in Positive Economics*, Chap. 1 (Chicago: University of Chicago Press, 1953). For an alternative view stressing the importance of using 'realistic' assumptions, see H. A. Simon, 'Rational Decision Making in Business Organizations', *American Economic Review*, 69, no. 4 (September 1979): 493–513.

[2] For an intermediate-level text containing an extensive set of real-world applications, see W. Nicholson and C. Snyder, *Intermediate Microeconomics and Its Application*, 11th ed. (Mason, OH: Thomson/Southwestern, 2010).

GENERAL FEATURES OF ECONOMIC MODELS

The number of economic models in current use is, of course, large. Specific assumptions used and the degree of detail provided vary greatly depending on the problem being addressed. The models used to explain the overall level of economic activity in South Africa, for example, must be considerably more aggregated and complex than those that seek to interpret the pricing of Western Cape oranges. Despite this variety, practically all economic models incorporate three common elements: (1) the *ceteris paribus* (other things the same) assumption; (2) the supposition that economic decision-makers seek to optimise something; and (3) a careful distinction between 'positive' and 'normative' questions. Because we will encounter these elements throughout this book, it may be helpful at the outset to describe the philosophy behind each of them.

The Ceteris Paribus Assumption

As in most sciences, models used in economics attempt to portray relatively simple relationships. A model of the market for wheat, for example, might seek to explain wheat prices with a small number of quantifiable variables, such as wages of farm workers, rainfall and consumer incomes. This parsimony in model specification permits the study of wheat pricing in a simplified setting in which it is possible to understand how the specific forces operate. Although any researcher will recognise that many 'outside' forces (e.g., presence of wheat diseases, changes in the prices of fertilisers or of tractors, or shifts in consumer attitudes about eating bread) affect the price of wheat, these other forces are held constant in the construction of the model. It is important to recognise that economists are not assuming that other factors do not affect wheat prices; rather, such other variables are assumed to be unchanged during the period of study. In this way, the effect of only a few forces can be studied in a simplified setting. Such ceteris paribus assumptions are used in all economic modelling.

Use of the ceteris paribus assumption does pose some difficulties for the verification of economic models from real-world data. In other sciences the problems may not be so severe because of the ability to conduct controlled experiments. For example, a physicist who wishes to test a model of the force of gravity probably would not do so by dropping objects from the Eiffel Tower. Experiments conducted in that way would be subject to too many extraneous forces (e.g., wind currents, particles in the air, variations in temperature) to permit a precise test of the theory. Rather, the physicist would conduct experiments in a laboratory, using a partial vacuum in which most other forces could be controlled or eliminated. In this way, the theory could be verified in a simple setting, without considering all the other forces that affect falling bodies in the real world.

With a few notable exceptions, economists have not been able to conduct controlled experiments to test their models. Instead, they have been forced to rely on various statistical methods to control for other forces when testing their theories. Although these statistical methods are as valid in principle as the controlled experiment methods used by other scientists, in practice they raise a number of thorny issues. For that reason, the limitations and precise meaning of the ceteris paribus assumption in economics are subject to greater controversy than in the laboratory sciences.

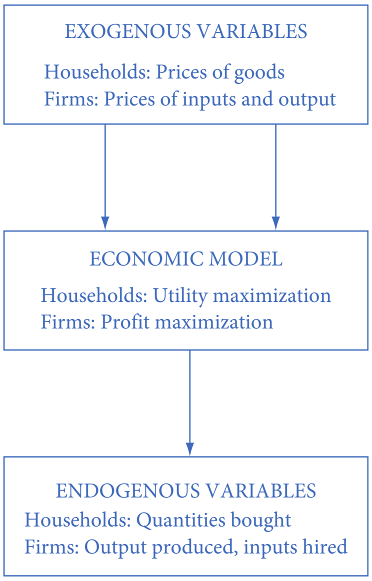
Structure of Economic Models

Most of the economic models you will encounter in this book will have a mathematical structure. They will highlight the relationships between factors that affect the decisions of households and firms and the results of those decisions. Economists tend to use different names for these two types of factors (or, in mathematical terms, *variables*). Variables that are outside of a decision-maker's control are called *exogenous variables*. Such variables are inputs into economic models. For example, in consumer theory we will usually treat individuals as price-takers. The prices of goods are determined outside of our models of consumer behaviour, and we wish to study how consumers adjust to them. The results of such decisions (e.g., the quantities of each good that a consumer buys) are *endogenous variables*. These variables are determined within our models. This distinction is pictured schematically in Figure 1.1. Although the actual models developed by economists may be complicated, they all have this basic structure. A good way to start studying a particular model is to identify precisely how it fits into this framework.

This distinction between exogenous and endogenous variables will become clearer as we explore a variety of economic models. Keeping straight which variables are determined outside a particular model and which

FIGURE 1.1

Structure of a Typical Microeconomic Model



variables are determined within a model can be confusing; therefore, we will try to remind you about this as we go along. The distinction between exogenous and endogenous variables is also helpful in understanding the way in which the ceteris paribus assumption is incorporated into economic models. In most cases we will want to study how the results of our models change when one of the exogenous variables changes. It is possible, even likely, that the change in such a single variable will change all the results calculated from the model. For example, as we will see, it is likely that the change in the price of a single good will cause an individual to change the quantities of practically every good he or she buys. Examining all such responses is precisely why economists build models. The ceteris paribus assumption is enforced by changing only one exogenous variable, holding all others constant. If we wish to study the effects of a change in the price of petrol on a household's purchases, we change that price in our model, but we do not change the prices of other goods (and in some cases we do not change the individual's income either). Holding the other prices constant is what is meant by studying the ceteris paribus effect of an increase in the price of petrol.

Optimisation Assumptions

Many economic models start from the assumption that the economic actors being studied are rationally pursuing some goal. We briefly discussed such an assumption when investigating the notion of firms maximising profits. Example 1.1 shows how that model can be used to make testable predictions. Other examples we will encounter in this book include consumers maximising their own well-being (utility), firms minimising costs and government regulators attempting to maximise public welfare. Although, as we will show, all these assumptions are unrealistic, and all have won widespread acceptance as good starting places for developing economic models. There seem to be two reasons for this acceptance. First, the optimisation assumptions are useful for generating precise, solvable models, primarily because such models can draw on a variety of mathematical techniques suitable for optimisation problems. Many of these techniques, together with the logic behind them, are reviewed in Chapter 2. A second reason for the popularity of optimisation models concerns their apparent empirical validity. Such models seem to be fairly good at explaining reality. In all, then, optimisation models have come to occupy a prominent position in modern economic theory.

EXAMPLE 1.1

Profit Maximisation

The profit-maximisation hypothesis provides a good illustration of how optimisation assumptions can be used to generate empirically testable propositions about economic behaviour. Suppose that a firm can sell all the output that it wishes at a price of p per unit and that the total costs of production, C, depend on the amount produced, q. Then profits are given by

$$\text{profits} = \pi = pq - C(q). \tag{1.1}$$

Maximisation of profits consists of finding that value of q which maximises the profit expression in Equation 1.1. This is a simple problem in calculus. Differentiation of Equation 1.1 and setting that derivative equal to 0 give the following first-order condition for a maximum:

$$\frac{d\pi}{dq} = p - C'(q) = 0 \quad \text{or} \quad p = C'(q). \tag{1.2}$$

In words, the profit-maximising output level (q^*) is found by selecting that output level for which price is equal to marginal cost, $C'(q)$. This result should be familiar to you from your introductory economics course. Notice that in this derivation the price for the firm's output is treated as a constant because the firm is a price-taker. That is, price is an exogenous variable in this model.

Equation 1.2 is only the first-order condition for a maximum. Taking account of the second-order condition can help us to derive a testable implication of this model. The second-order condition for a maximum is that at q^* it must be the case that

$$\frac{d^2\pi}{dq^2} = -C''(q) < 0 \quad \text{or} \quad C''(q^*) > 0. \tag{1.3}$$

That is, marginal cost must be increasing at q^* for this to be a true point of maximum profits.

Our model can now be used to 'predict' how a firm will react to a change in price. To do so, we differentiate Equation 1.2 with respect to price (p), assuming that the firm continues to choose a profit-maximising level of q:

$$\frac{d[p - C'(q^*) = 0]}{dp} = 1 - C''(q^*) \cdot \frac{dq^*}{dp} = 0. \tag{1.4}$$

Rearranging terms a bit gives

$$\frac{dq^*}{dp} = \frac{1}{C''(q^*)} > 0. \tag{1.5}$$

Here the final inequality again reflects the fact that marginal cost must be increasing at q^* if this point is to be a true maximum. This then is one of the testable propositions of the profit-maximisation hypothesis – if other things do not change, a price-taking firm should respond to an increase in price by increasing output. On the other hand, if firms respond to increases in price by reducing output, there must be something wrong with our model.

Although this is a simple model, it reflects the way we will proceed throughout much of this book. Specifically, the fact that the primary implication of the model is derived by calculus, and consists of showing what sign a derivative should have, is the kind of result we will see many times. Notice that in this model there is only one endogenous variable – q, the quantity the firm chooses to produce. There is also only one exogenous variable – p, the price of the product, which the firm takes as a given. Our model makes a specific prediction about how changes in this exogenous variable affect the firm's output choice.

QUERY: In general terms, how would the implications of this model be changed if the price a firm obtains for its output were a function of how much it sold? That is, how would the model work if the price-taking assumption were abandoned?

Positive–Normative Distinction

A final feature of most economic models is the attempt to differentiate carefully between 'positive' and 'normative' questions. Thus far we have been concerned primarily with positive economic theories. Such theories take the real world as an object to be studied, attempting to explain those economic phenomena that are observed. Positive economics seeks to determine how resources are *in fact* allocated in an economy. A somewhat different use of economic theory is *normative* analysis, taking a definite stance about what *should* be done. Under the heading of normative analysis, economists have a great deal to say about how resources *should* be allocated. For example, an economist engaged in positive analysis might investigate how prices are determined in the British National Health Service economy. The economist also might want to measure the costs and benefits of devoting even more resources to healthcare by, for example, offering government-subsidised health insurance. But when he or she specifically advocates that such an insurance plan should be adopted, the analysis becomes normative.

Some economists believe that the only proper economic analysis is positive analysis. Drawing an analogy with the physical sciences, they argue that 'scientific' economics should concern itself only with the description (and possibly prediction) of real-world economic events. To take political positions and to plead for special interests are considered to be outside the competence of an economist acting as such. Of course, an economist, like any other citizen, is free to express his or her views on political matters. But when doing so he or she is acting as a citizen, not an economist. For other economists, however, the positive-normative distinction seems artificial. They believe that the study of economics necessarily involves the researchers' own views about ethics, morality and fairness. According to these economists, searching for scientific 'objectivity' in such circumstances is hopeless. Despite some ambiguity, this book tries to adopt a positivist tone, leaving normative concerns for you to decide for yourself.

DEVELOPMENT OF THE ECONOMIC THEORY OF VALUE

Because economic activity has been a central feature of all societies, it is surprising that these activities were not studied in any detail until fairly recently. For the most part, economic phenomena were treated as a basic aspect of human behaviour that was not sufficiently interesting to deserve specific attention. It is, of course, true that individuals have always studied economic activities with a view toward making some kind of personal gain. Roman traders were not above making profits on their transactions. But investigations into the basic nature of these activities did not begin in any depth until the eighteenth century.[3] Because this book is about economic theory as it stands today, rather than the history of economic thought, our discussion of the evolution of economic theory will be brief. Only one area of economic study will be examined in its historical setting: the *theory of value*.

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[3] For a detailed treatment of early economic thought, see the classic work by J. A. Schumpeter, *History of Economic Analysis*, pt II, Chapters 1–3 (New York: Oxford University Press, 1954).

Early Economic Thoughts on Value

The theory of value, not surprisingly, concerns the determinants of the 'value' of a commodity. This subject is at the centre of modern microeconomic theory and is closely intertwined with the fundamental economic problem of allocating scarce resources to alternative uses. The logical place to start is with a definition of the word 'value'. Unfortunately, the meaning of this term has not been consistent throughout the development of the subject. Today we regard value as being synonymous with the price of a commodity.[4] Earlier philosopher-economists, however, made a distinction between the market price of a commodity and its value. The term value was then thought of as being, in some sense, synonymous with 'importance,' 'essentiality' or (at times) 'godliness'. Because 'price' and 'value' were separate concepts, they could differ, and most early economic discussions centred on these divergences. For example, St Thomas Aquinas believed value to be divinely determined. Because prices were set by humans, it was possible for the price of a commodity to differ from its value. A person accused of charging a price in excess of a good's value was guilty of charging an 'unjust' price. St Thomas believed that, in most cases, the 'just' rate of interest was zero. Any lender who demanded a payment for the use of money was charging an unjust price and could be – and sometimes was – prosecuted by Church officials.

The Founding of Modern Economics

During the latter part of the eighteenth century, philosophers began to take a more scientific approach to economic questions. The 1776 publication of *The Wealth of Nations* by Adam Smith (1723–1790) is generally considered the beginning of modern economics. In his vast, all-encompassing work, Smith laid the foundation for thinking about market forces in an ordered and systematic way. Still, Smith and his immediate successors, such as David Ricardo (1772–1823), continued to distinguish between value and price. To Smith, for example, the value of a commodity meant its 'value in use', whereas the price represented its 'value in exchange'. The distinction between these two concepts was illustrated by the famous water–diamond paradox. Water, which obviously has great value in use, has little value in exchange (it has a low price); diamonds are of little practical use but have a great value in exchange. The paradox with which early economists struggled derives from the observation that some useful items have low prices whereas certain non-essential items have high prices.

Labour Theory of Exchange Value

Neither Smith nor Ricardo ever satisfactorily resolved the water–diamond paradox. The concept of value in use was left for philosophers to debate, while economists turned their attention to explaining the determinants of value in exchange (i.e., to explaining relative prices). One obvious possible explanation is that exchange values of goods are determined by what it costs to produce them. Costs of production are primarily influenced by labour costs – at least this was so in the time of Smith and Ricardo – and therefore it was a short step to embrace a labour theory of value. For example, to paraphrase an example from Smith, if catching a deer takes twice the number of labour hours as catching a beaver, then one deer should exchange for two beavers. In other words, the price of a deer should be twice that of a beaver. Similarly, diamonds are relatively costly because their production requires substantial labour input, whereas water is freely available.

To students with even a passing knowledge of what we now call the law of *supply and demand*, Smith's and Ricardo's explanation must seem incomplete. Did they not recognise the effects of demand on price? The answer to this question is both yes and no. They did observe periods of rapidly rising and falling relative prices and attributed such changes to demand shifts. However, they regarded these changes as abnormalities that produced only a temporary divergence of market price from labour value. Because they had not really developed a theory of value in use, they were unwilling to assign demand any more than a transient role in determining relative prices. Rather, long-run exchange values were assumed to be determined solely by labour costs of production.

The Marginalist Revolution

Between 1850 and 1880, economists became increasingly aware that to construct an adequate alternative to the labour theory of value, they had to devise a theory of value in use. During the 1870s, several economists discovered that it is

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[4] This is not completely true when 'externalities' are involved, and a distinction must be made between private and social value (see Chapter 19).

not the total usefulness of a commodity that helps to determine its exchange value, but rather the usefulness of the *last unit consumed*. For example, water is certainly useful – it is necessary for all life. However, because water is relatively plentiful, consuming one more litre (ceteris paribus) has a relatively low value to people. These 'marginalists' redefined the concept of value in use from an idea of overall usefulness to one of marginal, or incremental, usefulness – the usefulness of an *additional unit of a commodity*. The concept of the demand for an incremental unit of output was now contrasted with Smith's and Ricardo's analysis of production costs to derive a comprehensive picture of price determination.[5]

Marshallian Supply–Demand Synthesis

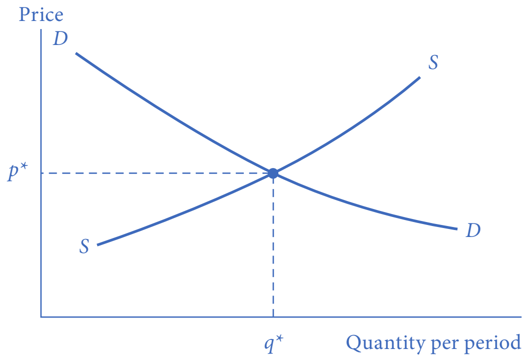
The clearest statement of these marginal principles was presented by the English economist Alfred Marshall (1842–1924) in his *Principles of Economics*, published in 1890. Marshall showed that demand and supply *simultaneously* operate to determine price. As Marshall noted, just as you cannot tell which blade of a scissors does the cutting, so too you cannot say that either demand or supply alone determines price. That analysis is illustrated by the famous Marshallian cross shown in Figure 1.2. In the diagram the quantity of a good purchased per period is shown on the horizontal axis, and its price appears on the vertical axis. The curve *DD* represents the quantity of the good demanded per period at each possible price. The curve is negatively sloped to reflect the marginalist principle that, as quantity increases, people are willing to pay less for the last unit purchased. It is the value of this last unit that sets the price for all units purchased. The curve *SS* shows how (marginal) production costs increase as more output is produced. This reflects the increasing cost of producing one more unit as total output expands. In other words, the upward slope of the *SS* curve reflects increasing marginal costs, just as the downward slope of the *DD* curve reflects decreasing marginal value. The two curves intersect at p^*, q^*. This is an *equilibrium* point – both buyers and sellers are content with the quantity being traded and the price at which it is traded. If one of the curves should shift, the equilibrium point would shift to a new location. Thus, price and quantity are simultaneously determined by the joint operation of supply and demand.

Paradox Resolved

Marshall's model resolves the water–diamond paradox. Prices reflect both the marginal evaluation that demanders place on goods and the marginal costs of producing the goods. Viewed in this way, there is no paradox. Water is low in price because it has both a low marginal value and a low marginal cost of production. On the other hand, diamonds are high in price because they have both a high marginal value (because people are willing to pay quite a bit for one more) and a high marginal cost of production. This basic model of supply and demand lies behind much of the analysis presented in this book.

FIGURE 1.2

The Marshallian Supply–Demand Cross



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[5] Ricardo had earlier provided an important first step in marginal analysis in his discussion of rent. Ricardo theorised that as the production of corn increased, land of inferior quality would be used and this would cause the price of corn to increase. In his argument Ricardo recognised that it is the marginal cost – the cost of producing an additional unit – that is relevant to pricing. Notice that Ricardo implicitly held other inputs constant when discussing decreasing land productivity; that is, he used one version of the ceteris paribus assumption.

EXAMPLE 1.2

Supply–Demand Equilibrium

Although graphical presentations are adequate for some purposes, economists often use algebraic representations of their models both to clarify their arguments and to make them more precise. As an elementary example, suppose we wished to study the market for peanuts and, based on the statistical analysis of historical data, concluded that the quantity of peanuts demanded each week (q, measured in bushels) depended on the price of peanuts (p, measured in Euros per bushel) according to the equation:

$$\text{quantity demanded} = q_D = 1000 - 100p. \tag{1.6}$$

Because this equation for q_D contains only the single independent variable p, we are implicitly holding constant all other factors that might affect the demand for peanuts. Equation 1.6 indicates that, if other things do not change, at a price of €5 per bushel people will demand 500 bushels of peanuts, whereas at a price of €4 per bushel they will demand 600 bushels. The negative coefficient for p in Equation 1.6 reflects the marginalist principle that a lower price will cause people to buy more peanuts.

To complete this simple model of pricing, suppose that the quantity of peanuts supplied also depends on price:

$$\text{quantity supplied} = q_S = -125 + 125p. \tag{1.7}$$

Here the positive coefficient of price also reflects the marginal principle that a higher price will call forth increased supply – primarily because (as we saw in Example 1.1) it permits firms to incur higher marginal costs of production without incurring losses on the additional units produced.

Equilibrium Price Determination

Therefore, Equations 1.6 and 1.7 reflect our model of price determination in the market for peanuts. An equilibrium price can be found by setting quantity demanded equal to quantity supplied:

$$q_D = q_S \tag{1.8}$$

or

$$1000 - 100p = -125 + 125p \tag{1.9}$$

or

$$225p = 1125 \tag{1.10}$$

thus,

$$p^* = 5. \tag{1.11}$$

At a price of €5 per bushel, this market is in equilibrium: At this price people want to purchase 500 bushels, and that is exactly what peanut producers are willing to supply. This equilibrium is pictured graphically as the intersection of D and S in Figure 1.3.

A More General Model

To illustrate how this supply–demand model might be used, let's adopt a more general notation. Suppose now that the demand and supply functions are given by

$$q_D = a + bp \quad \text{and} \quad q_S = c + dp \tag{1.12}$$

where a and c are constants that can be used to shift the demand and supply curves, respectively, and b (< 0) and d (> 0) represent demanders' and suppliers' reactions to price. Equilibrium in this market requires

$$\begin{gathered} q_D = q_S \quad \text{or} \\ a + bp = c + dp. \end{gathered} \tag{1.13}$$

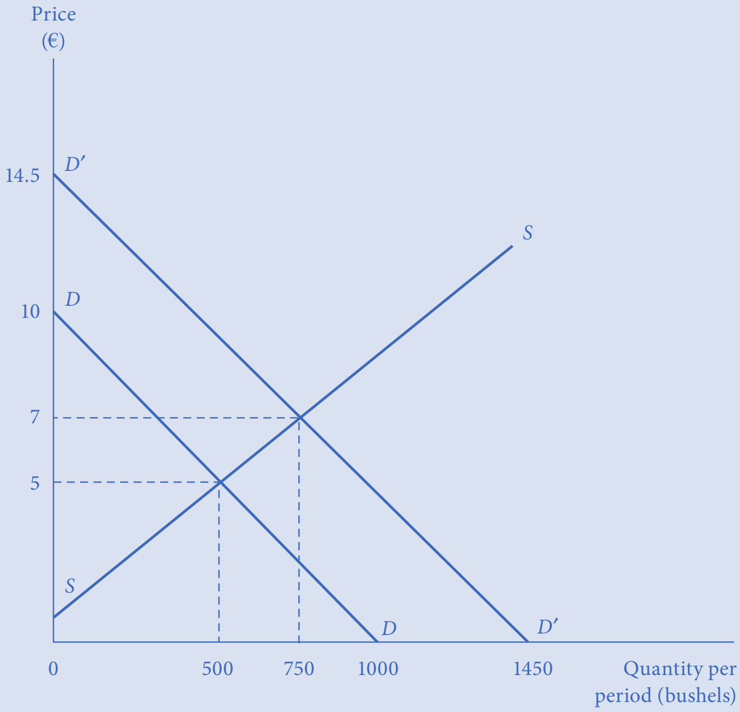
Thus, equilibrium price is given by[6]

$$p^* = \frac{a - c}{d - b}. \tag{1.14}$$

FIGURE 1.3

Changing Supply–Demand Equilibria

The initial supply–demand equilibrium is illustrated by the intersection of D and S ($p^* = 5$, $q = 500$). When demand shifts to $qD' = 1450 - 100p$ (denoted as D'), the equilibrium shifts to $p^* = 7$, $q^* = 750$.



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[6] Equation 1.14 is sometimes called the 'reduced form' for the supply–demand structural model of Equations 1.12 and 1.13. It shows that the equilibrium value for the endogenous variable p ultimately depends only on the exogenous factors in the model (a and c) and on the behavioural parameters b and d. A similar equation can be calculated for equilibrium quantity.

Notice that in our previous example $a = 1000$, $b = -100$, $c = -125$, and $d = 125$; therefore,

$$p^* = \frac{1000 + 125}{125 + 100} = \frac{1125}{225} = 5. \tag{1.15}$$

With this more general formulation, however, we can pose questions about how the equilibrium price might change if either the demand or supply curve shifted. For example, differentiation of Equation 1.14 shows that

$$\begin{aligned} \frac{dp^*}{da} &= \frac{1}{d-b} > 0, \\ \frac{dp^*}{dc} &= \frac{-1}{d-b} < 0. \end{aligned} \tag{1.16}$$

That is, an increase in demand (an increase in a) increases equilibrium price, whereas an increase in supply (an increase in c) reduces price. This is exactly what a graphical analysis of supply and demand curves would show. For example, Figure 1.3 shows that when the constant term, a, in the demand equation increases to 1450, equilibrium price increases to $p^* = 7$ [$= (1450 + 125)/225$].

QUERY: How might you use Equation 1.16 to 'predict' how each unit increase in the exogenous constant a affects the endogenous variable p^*? Does this equation correctly predict the increase in p^* when the constant a increases from 1000 to 1450?

General Equilibrium Models

Although the Marshallian model is an extremely useful and versatile tool, it is a *partial equilibrium model*, looking at only one market at a time. For some questions, this narrowing of perspective gives valuable insights and analytical simplicity. For other, broader questions, such a narrow viewpoint may prevent the discovery of important relationships among markets. To answer more general questions we must have a model of the whole economy that suitably mirrors the connections among various markets and economic agents. The French economist Leon Walras (1831–1910), building on a long Continental tradition in such analysis, created the basis for modern investigations into those broad questions. His method of representing the economy by a large number of simultaneous equations forms the basis for understanding the interrelationships implicit in *general equilibrium* analysis. Walras recognised that one cannot talk about a single market in isolation; what is needed is a model that permits the effects of a change in one market to be followed through other markets.

For example, suppose that the demand for peanuts were to increase. This would cause the price of peanuts to increase. Marshallian analysis would seek to understand the size of this increase by looking at conditions of supply and demand in the peanut market. General equilibrium analysis would look not only at that market but also at repercussions in other markets. An increase in the price of peanuts would increase costs for peanut butter makers, which would, in turn, affect the supply curve for peanut butter. Similarly, the increasing price of peanuts might mean higher land prices for peanut farmers, which would affect the demand curves for all products that they buy. The demand curves for cars, furniture and trips to Thailand would all shift out, and that might create additional incomes for the providers of those products. Consequently, the effects of the initial increase in demand for peanuts eventually would spread throughout the economy. General equilibrium analysis attempts to develop models that permit us to examine such effects in a simplified setting. Several models of this type are described in Chapter 13.

Production Possibility Frontier

Here we briefly introduce some general equilibrium ideas by using another graph you should remember from introductory economics – the *production possibility frontier*. This graph shows the various amounts of two goods that an economy can produce using its available resources during some period (say, one week). Because the production possibility frontier shows two goods, rather than the single good in Marshall's model, it is used as a basic building block for general equilibrium models.

Figure 1.4 shows the production possibility frontier for two goods: food and clothing. The graph illustrates the supply of these goods by showing the combinations that can be produced with this economy's resources. For example, 10 kilograms of food and 3 units of clothing could be produced, or 4 kilograms of food and 12 units of clothing. Many other combinations of food and clothing could also be produced. The production possibility frontier shows all of them. Combinations of food and clothing outside the frontier cannot be produced because not enough resources are available. The production possibility frontier reminds us of the basic economic fact that resources are scarce – there are not enough resources available to produce all we might want of every good.

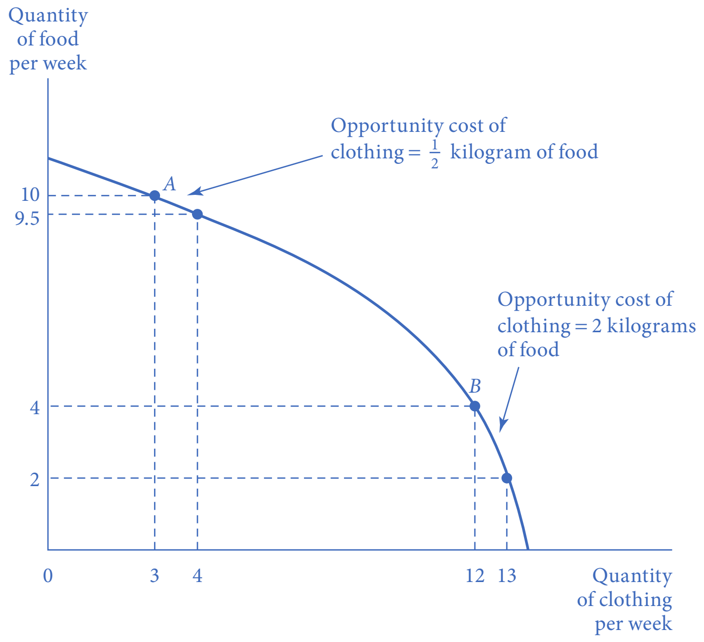
This scarcity means that we must choose how much of each good to produce. Figure 1.4 makes clear that each choice has its costs. For example, if this economy produces 10 kilograms of food and 3 units of clothing at point *A*, producing 1 more unit of clothing would 'cost' ½ kilogram of food – increasing the output of clothing by 1 unit means the production of food would have to decrease by ½ kilogram. Thus, the *opportunity cost* of 1 unit of clothing at point *A* is ½ kilogram of food. On the other hand, if the economy initially produces 4 kilograms of food and 12 units of clothing at point *B*, it would cost 2 kilograms of food to produce 1 more unit of clothing. The opportunity cost of 1 more unit of clothing at point *B* has increased to 2 kilograms of food. Because more units of clothing are produced at point *B* than at point *A*, both Ricardo's and Marshall's ideas of increasing incremental costs suggest that the opportunity cost of an additional unit of clothing will be higher at point *B* than at point *A*. This effect is shown by Figure 1.4.

The production possibility frontier provides two general equilibrium insights that are not clear in Marshall's supply and demand model of a single market. First, the graph shows that producing more of one good means producing less of another good because resources are scarce. Economists often (perhaps too often!) use the expression 'there is no such thing as a free lunch' to explain that every economic action has opportunity costs. Second, the production possibility frontier shows that opportunity costs depend on how much of each good is produced. The frontier is like a supply curve for two goods: It shows the opportunity cost of producing more of one good as the decrease in the amount of the second good. Therefore, the production possibility frontier is a particularly useful tool for studying several markets at the same time.

FIGURE 1.4

Production Possibility Frontier

The production possibility frontier shows the different combinations of two goods that can be produced from a certain amount of scarce resources. It also shows the opportunity cost of producing more of one good as the amount of the other good that cannot then be produced. The opportunity cost at two different levels of clothing production can be seen by comparing points *A* and *B*.



EXAMPLE 1.3

The Production Possibility Frontier and Economic Inefficiency

General equilibrium models are good tools for evaluating the efficiency of various economic arrangements. As we will see in Chapter 13, such models have been used to assess a wide variety of policies such as trade agreements, tax structures and environmental regulations. In this simple example, we explore the idea of efficiency in its most elementary form.

Suppose that an economy produces two goods, x and y, using labour as the only input. The production function for good x is $x = l_x^{0.5}$ (where l_x is the quantity of labour used in x production), and the production function for good y is $y = 2l_y^{0.5}$. Total labour available is constrained by $l_x + l_y \leq 200$. Construction of the production possibility frontier in this economy is extremely simple:

$$l_x + l_y = x^2 + 0.25y^2 \leq 200 \tag{1.17}$$

where the equality holds exactly if the economy is to be producing as much as possible (which, after all, is why it is called a 'frontier'). Equation 1.17 shows that the frontier here has the shape of a quarter ellipse – its concavity derives from the diminishing returns exhibited by each production function.

Opportunity Cost

Assuming this economy is on the frontier, the opportunity cost of good y in terms of good x can be derived by solving for y as

$$y^2 = 800 - 4x^2 \quad \text{or} \quad y = \sqrt{800 - 4x^2} = [800 - 4x^2]^{0.5} \tag{1.18}$$

And then differentiating this expression:

$$\frac{dy}{dx} = 0.5[800 - 4x^2]^{-0.5}(-8x) = \frac{-4x}{y}. \tag{1.19}$$

Suppose, for example, labour is equally allocated between the two goods. Then $x = 10$, $y = 20$ and $dy/dx = -4(10)/20 = -2$. With this allocation of labour, each unit increase in x output would require a reduction in y of 2 units. This can be verified by considering a slightly different allocation, $l_x = 101$ and $l_y = 99$. Now production is $x = 10.05$ and $y = 19.9$. Moving to this alternative allocation would have

$$\frac{\Delta y}{\Delta x} = \frac{(19.9 - 20)}{(10.05 - 10)} = \frac{-0.1}{0.05} = -2,$$

which is precisely what was derived from the calculus approach.

Concavity

Equation 1.19 clearly illustrates the concavity of the production possibility frontier. The slope of the frontier becomes steeper (more negative) as x output increases and y output decreases. For example, if labour is allocated so that $l_x = 144$ and $l_y = 56$, then outputs are $x = 12$ and $y \approx 15$ and so $dy/dx = -4(12)/15 = -3.2$. With expanded x production, the opportunity cost of one more unit of x increases from 2 to 3.2 units of y.

Inefficiency

If an economy operates inside its production possibility frontier, it is operating inefficiently. Moving outward to the frontier could increase the output of both goods. In this book we will explore many reasons for such inefficiency. These usually derive from a failure of some market to perform correctly. For the purposes of this illustration, let's assume that the labour market in this economy does not work well and that 20 workers are permanently unemployed. Now the production possibility frontier becomes

$$x^2 + 0.25y^2 = 180, \tag{1.20}$$

and the output combinations we described previously are no longer feasible. For example, if $x = 10$, then y output is now $y \approx 17.9$. The loss of approximately 2.1 units of y is a measure of the cost of the labour market inefficiency. Alternatively, if the labour supply of 180 were allocated evenly between the production of the two goods, then we would have $x \approx 9.5$ and $y \approx 19$, and the inefficiency would show up in both goods' production – more of both goods could be produced if the labour market inefficiency were resolved.

QUERY: How would the inefficiency cost of labour market imperfections be measured solely in terms of x production in this model? How would it be measured solely in terms of y production? What would you need to know to assign a single number to the efficiency cost of the imperfection when labour is equally allocated to the two goods?

Welfare Economics

In addition to using economic models to examine positive questions about how the economy operates, the tools used in general equilibrium analysis have also been applied to the study of normative questions about the welfare properties of various economic arrangements. Although such questions were a major focus of the great eighteenth- and nineteenth-century economists (e.g., Smith, Ricardo, Marx and Marshall), perhaps the most significant advances in their study were made by the British economist Francis Y. Edgeworth (1848–1926) and the Italian economist Vilfredo Pareto (1848–1923) in the early years of the twentieth century. These economists helped to provide a precise definition for the concept of 'economic efficiency' and to demonstrate the conditions under which markets will be able to achieve that goal. By clarifying the relationship between the allocation pricing of resources, they provided some support for the idea, first enunciated by Adam Smith, that properly functioning markets provide an 'invisible hand' that helps allocate resources efficiently. Later sections of this book focus on some of these welfare issues.

MODERN DEVELOPMENTS

Research activity in economics expanded rapidly in the years following World War II. A major purpose of this book is to summarise much of this research. By illustrating how economists have tried to develop models to explain increasingly complex aspects of economic behaviour, this book seeks to help you recognise some of the remaining unanswered questions.

The Mathematical Foundations of Economic Models

A major post-war development in microeconomic theory was the clarification and formalisation of the basic assumptions that are made about individuals and firms. The first landmark in this development was the 1947 publication of Paul Samuelson's *Foundations of Economic Analysis*, in which the author (the first American Nobel Prize winner in economics) laid out a number of models of optimising behaviour.[7] Samuelson demonstrated the importance of

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[7] Paul A. Samuelson, *Foundations of Economic Analysis* (Cambridge, MA: Harvard University Press, 1947).

basing behavioural models on well-specified mathematical postulates so that various optimisation techniques from mathematics could be applied. The power of his approach made it inescapably clear that mathematics had become an integral part of modern economics. In Chapter 2 of this book we review some of the mathematical concepts most often used in microeconomics.

New Tools for Studying Markets

A second feature that has been incorporated into this book is the presentation of a number of new tools for explaining market equilibria. These include techniques for describing pricing in single markets, such as increasingly sophisticated models of monopolistic pricing or models of the strategic relationships among firms that use game theory. They also include general equilibrium tools for simultaneously exploring relationships among many markets. As we shall see, all these new techniques help to provide a more complete and realistic picture of how markets operate.

The Economics of Uncertainty and Information

A final major theoretical advance during the post-war period was the incorporation of uncertainty and imperfect information into economic models. Some of the basic assumptions used to study behaviour in uncertain situations were originally developed in the 1940s in connection with the theory of games. Later developments showed how these ideas could be used to explain why individuals tend to be adverse to risk and how they might gather information to reduce the uncertainties they face. In this book, problems of uncertainty and information enter the analysis on many occasions.

Computers and Empirical Analysis

One final aspect of the post-war development of microeconomics should be mentioned – the increasing use of computers to analyse economic data and build economic models. As computers have become able to handle larger amounts of information and carry out complex mathematical manipulations, economists' ability to test their theories has dramatically improved. Whereas previous generations had to be content with rudimentary tabular or graphical analyses of real-world data, today's economists have available a wide variety of sophisticated techniques together with extensive microeconomic data with which to test their models. To examine these techniques and some of their limitations would be beyond the scope and purpose of this book.

SUMMARY

This chapter provided background on how economists approach the study of the allocation of resources. Much of the material discussed here should be familiar to you from introductory economics. In many respects, the study of economics represents acquiring increasingly sophisticated tools for addressing the same basic problems. The purpose of this book (and, indeed, of most upper-level books on economics) is to provide you with more of these tools. As a starting place, this chapter reminded you of the following points:

- Economics is the study of how scarce resources are allocated among alternative uses. Economists seek to develop simple models to help understand that process. Many of these models have a mathematical basis because the use of mathematics offers a precise shorthand for stating the models and exploring their consequences.
- The most commonly used economic model is the supply–demand model first thoroughly developed by Alfred Marshall in the latter part of the nineteenth century. This model shows how observed prices can be taken to represent an equilibrium balancing of the production costs incurred by firms and the willingness of demanders to pay for those costs.
- Marshall's model of equilibrium is only 'partial' – that is, it looks only at one market at a time. To look at many markets together requires an expanded set of general equilibrium tools.
- Testing the validity of an economic model is perhaps the most difficult task economists face. Occasionally, a model's validity can be appraised by asking whether it is based on 'reasonable' assumptions. More often, however, models are judged by how well they can explain economic events in the real world.

2

MATHEMATICS FOR MICROECONOMICS

Microeconomic models are constructed using a wide variety of mathematical techniques. In this chapter we provide a brief summary of some of the most important techniques that you will encounter in this book. A major portion of the chapter concerns mathematical procedures for finding the optimal value of some function. Because we will frequently adopt the assumption that an economic actor seeks to maximise or minimise some function, we will encounter these procedures (most of which are based on calculus) many times.

After our detailed discussion of the calculus of optimisation, we turn to four topics that are covered more briefly. First, we look at a few special types of functions that arise in economics. Knowledge of properties of these functions can often be helpful in solving problems. Next, we provide a brief summary of integral calculus. Although integration is used in this book far less frequently than is differentiation, we will nevertheless encounter situations where we will want to use integrals to measure areas that are important to economic theory or to add up outcomes that occur over time or across many individuals. One particular use of integration is to examine problems in which the objective is to maximise a stream of outcomes over time. Our third added topic focuses on techniques to be used for such problems in dynamic optimisation. Finally, Chapter 2 concludes with a brief summary of mathematical statistics, which will be particularly useful in our study of economic behaviour in uncertain situations.

MAXIMISATION OF A FUNCTION OF ONE VARIABLE

We can motivate our study of optimisation with a simple example. Suppose that a manager of a firm desires to maximise[1] the profits received from selling a particular good. Suppose also that the profits (π) received depend only on the quantity (q) of the good sold. Mathematically,

$$\pi = f(q). \tag{2.1}$$

Figure 2.1 shows a possible relationship between π and q. Clearly, to achieve maximum profits, the manager should produce output q^*, which yields profits π^*. If a graph such as that of Figure 2.1 were available, this would seem to be a simple matter to be accomplished with a ruler.

Suppose, however, as is more likely, the manager does not have such an accurate picture of the market. He or she may then try varying q to see where a maximum profit is obtained. For example, by starting at q_1, profits from sales would be π_1. Next, the manager may try output q_2, observing that profits have increased to π_2. The commonsense idea that profits have increased in response to an increase in q can be stated formally as

$$\frac{\pi_2 - \pi_1}{q_2 - q_1} > 0 \quad \text{or} \quad \frac{\Delta\pi}{\Delta q} > 0, \tag{2.2}$$

where the Δ notation is used to mean 'the change in' π or q. As long as $\Delta\pi/\Delta q$ is positive, profits are increasing and the manager will continue to increase output. For increases in output to the right of q^*, however, $\Delta\pi/\Delta q$ will be negative, and the manager will realise that a mistake has been made.

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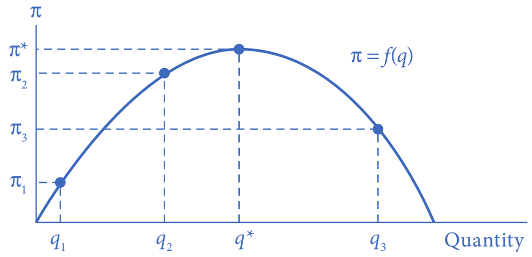
[1] Here we will generally explore maximisation problems. A virtually identical approach would be taken to study minimisation problems because maximisation of $f(x)$ is equivalent to minimising $-f(x)$.

FIGURE 2.1

Hypothetical Relationship between Quantity Produced and Profits

If a manager wishes to produce the level of output that maximises profits, then q^* should be produced. Notice that at q^*, $d\pi/dq = 0$.



Derivatives

As you probably know, the limit of $\Delta\pi/\Delta q$ for small changes in q is called the derivative of the function, $\pi = f(q)$, and is denoted by $d\pi/dq$ or df/dq or $f'(q)$. More formally, the derivative of a function $\pi = f(q)$ at the point q_1 is defined as

$$\frac{d\pi}{dq} = \frac{df}{dq} = \lim_{h\to 0} \frac{f(q_1 + h) - f(q_1)}{h}. \tag{2.3}$$

Notice that the value of this ratio obviously depends on the point q_1 that is chosen. The derivative of a function may not always exist or it may be undefined at certain points. Most of the functions studied in this book are fully differentiable, however.

Value of the Derivative at a Point

A notational convention should be mentioned: sometimes we wish to note explicitly the point at which the derivative is to be evaluated. For example, the evaluation of the derivative at the point $q = q_1$ could be denoted by

$$\left.\frac{d\pi}{dq}\right|_{q=q_1}. \tag{2.4}$$

At other times, we are interested in the value of $d\pi/dq$ for all possible values of q, and no explicit mention of a particular point of evaluation is made.

In the example of Figure 2.1,

$$\left.\frac{d\pi}{dq}\right|_{q=q_1} > 0,$$

whereas

$$\left.\frac{d\pi}{dq}\right|_{q=q_3} < 0.$$

What is the value of $d\pi/dq$ at q^*? It would seem to be 0 because the value is positive for values of q less than q^* and negative for values of q greater than q^*. The derivative is the slope of the curve in question; this slope is positive to the left of q^* and negative to the right of q^*. At the point q^*, the slope of $f(q)$ is 0.

First-Order Condition for a Maximum

This result is general. For a function of one variable to attain its maximum value at some point, the derivative at that point (if it exists) must be 0. Hence, if a manager could estimate the function $f(q)$ from some sort of real-world data, it would theoretically be possible to find the point where $df/dq = 0$. At this optimal point (say, q^*),

$$\left.\frac{df}{dq}\right|_{q=q^*} = 0. \tag{2.5}$$

Second-Order Conditions

An unsuspecting manager could be tricked, however, by a naive application of this first-derivative rule alone. For example, suppose that the profit function looks like that shown in either Figure 2.2(a) or (b). If the profit function is that shown in (a), the manager, by producing where $d\pi/dq = 0$, will choose point q_a^*. This point in fact yields minimum, not maximum, profits for the manager. Similarly, if the profit function is that shown in (b), the manager will choose point q_b^*, which, although it yields a profit greater than that for any output lower than q_b^*, is certainly inferior to any output greater than q_b^*. These situations illustrate the mathematical fact that $d\pi/dq = 0$ is a *necessary* condition for a maximum, but not a *sufficient* condition. To ensure that the chosen point is indeed a maximum point, a second condition must be imposed.

Intuitively, this additional condition is clear: The profit available by producing either a bit more or a bit less than q^* must be smaller than that available from q^*. If this is not true, the manager can do better than q^*. Mathematically, this means that $d\pi/dq$ must be greater than 0 for $q < q^*$ and must be less than 0 for $q > q^*$. Therefore, at q^*, $d\pi/dq$ must be decreasing. Another way of saying this is that the derivative of $d\pi/dq$ must be negative at q^*.

Second Derivatives

The derivative of a derivative is called a *second derivative* and is denoted by

$$\frac{d^2\pi}{dq^2} \quad \text{or} \quad \frac{d^2 f}{dq^2} \quad \text{or} \quad f''(q).$$

The additional condition for q^* to represent a (local) maximum is therefore where the notation is again a reminder that this second derivative is to be evaluated at q^*.

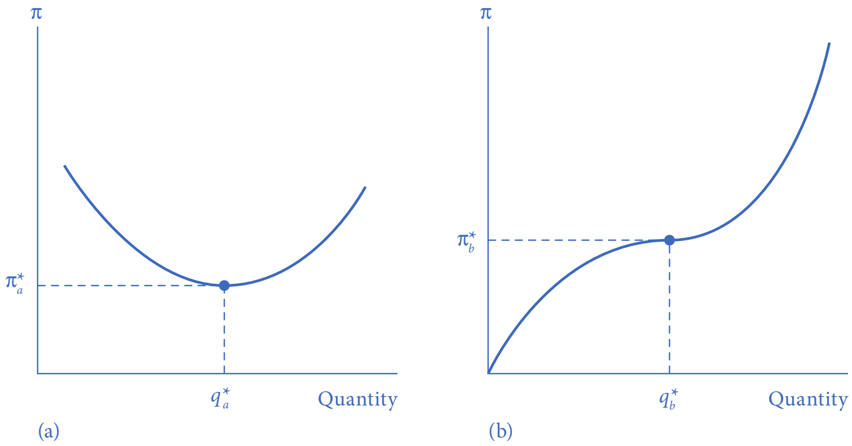
$$\left.\frac{d^2\pi}{dq^2}\right|_{q=q^*} = f''(q)\Big|_{q=q^*} < 0, \tag{2.6}$$

Hence although Equation 2.5 ($d\pi/dq = 0$) is a necessary condition for a maximum, that equation must be combined with Equation 2.6 ($d^2\pi/dq^2 < 0$) to ensure that the point is a local maximum for the function. Therefore, Equations 2.5 and 2.6 together are sufficient conditions for such a maximum. Of course, it is possible that by a series of trials the manager may be able to decide on q^* by relying on market information rather than on mathematical reasoning (remember Friedman's pool-player analogy). In this book we shall be less interested in how the point is discovered than in its properties and how the point changes when conditions change. A mathematical development will be helpful in answering these questions.

FIGURE 2.2

Two Profit Functions that Give Misleading Results if the First Derivative Rule is Applied Uncritically

In (a), the application of the first derivative rule would result in point q_a^* being chosen. This point is in fact a point of minimum profits. Similarly, in (b), output level q_b^* would be recommended by the first derivative rule, but this point is inferior to all outputs greater than q_b^*. This demonstrates graphically that finding a point at which the derivative is equal to 0 is a necessary, but not a sufficient, condition for a function to attain its maximum value.



(a)

(b)